HW5

1 Probability Axioms

- 1. Let A,B,C be three arbitrary events. Find the probability of exactly one of these events occuring.
- 2. If P(A) > 0, P(B) > 0, and P(A) < P(A|B), show that P(B) < P(B|A)

2 Simple Counting

- 1. Cards are dealt, one at a time, from a standard 52-card deck.
 - (a) If the first 2 cards are both spades, what is the probability that the next 3 cards are also spades?
 - (b) If the first 3 cards are all spades, what is the probability that the next 2 cards are also spades?
 - (c) If the first 4 cards are all spades, what is the probability that the next card is also a spade?
- 2. Five cards are drawn from a standard 52-card playing deck. What is the probability that all 5 cards will be of the same suit?
- 3. If two balanced dice are rolled, what the is the probability that the difference between the two numbers that appear will be less than 3?

3 Independence

- 1. Suppose that A and B are mutually exclusive events, with P(A) > 0 and P(B) < 1. Are A and B independent? Prove your answer.
- 2. Suppose that $A \subset B$ and that P(A) > 0 and P(B) > 0. Are A and B independent? Prove your answer.
- 3. Suppose that a balanced coin is independently tossed two times. Define the following events:
 - (a) Head appears on the first toss.
 - (b) Head appears on the second toss

(c) Both tosses yield the same outcome

Are A, B, and C mutually independent?

4 Combinatorics, Bayes

- 1. If 2n teams are to be assigned to games 1, 2, . . . , n, in how many ways can the teams be assigned to the games?
- 2. There is a box with two fair (equal probability of each side) coins in it. One coin has heads on both sides, the other one has heads and tails like a regular coin. You pick a coin randomly from the box and without looking at it, spin it and observe heads on the top. What is the probability that the other side is also heads?
- 3. If n letters are placed at random in n envelopes, what is the probability that exactly n-1 letters will be placed in the correct envelopes?
- 4. If A and B are mutually exclusive events and P(B) > 0, show that $P(A/A \cup B) = \frac{P(A)}{P(A) + P(B)}$.

5 Conditional Probability

- 1. We have two dice (of 6 faces). Let the outcomes of the dice be X,Y and let A = X+Y, $B = \max(X,Y)$. Find the conditional probability of A, given that B is even.
- 2. Suppose that a fair coin is tossed independently n times. Determine the probability of obtaining exactly n-1 heads, given (a) that at least n-2 heads are obtained and (b) that heads are obtained on the first n-2 tosses.
- 3. Three palyers A,B,C take turns tossing a fair coin. Suppose that A tosses first, B tosses second and C tosses third. This cycle is repeated indefinitely until someone wins by being the first player to obtain a head. Determine the probability that each of three players will win.