Real Analysis Key Concepts

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According to Principles of Mathematical Analysis by Walter Rudin (Chapter 1-5)

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Overview

Set Operations

Fields

Addition Axioms

Mutiplication Axioms

Boundaries

Upper Bound

Least Upper Bound

Lower Bound

Greatest Lower Bound

Dedekind Cut

1. non-trival
2. closed downwards
3. no largest number

eg: $\alpha = \mathbb{Q}^-$ Yes
$A = \{x | x^2 < 2, x \in \mathbb{Q}\}$ No

Numbers

Real Numbers ($\mathbb{R}$)

Relational Numbers ($\mathbb{Q}$)

Natural Numbers ($\mathbb{N}$)

Relationship bt. $\mathbb{R}$ and $\mathbb{Q}$

$\mathbb{C}$ Operations

Conjugate

Inner Product

Induction

1. Base case

2. Inductive steps

Sets Relationship

Injection

Surjection

Bijection

Finite Set

J_n

Finite sets:

Infinite sets: $\mathbb{N}$

Countable Set

Countable sets: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$

Uncountable sets: $\mathbb{R}$

Countability Theorems

Cantor Theorem

Powerset

Cantor Theorem: $A \not\approx 2^A$

Metric Spaces

1. non-negativity
2. symmetry
3. triangle inequality

eg: Euclidean spaces

Limit Point

Open ball

Closed ball

Limit point

Points and Sets

Isolated points

Interior points

Open set

Closed set

Closure

Closure of a set

Closure property:

Relationship bt. Open and Closed Sets

Compact Sets

Open Cover

Subcove

Compact Set

Bounded Sets

Compactness Theorems I

Compactness Theorems II

In R^n, K is compact $\Leftrightarrow$ K is closed and bounded.

Finite Interesction Property

Cantor Set

Connectness

Separated sets

Connected sets

eg: [a,b] is connected

Converge

Diverge

Converge Theorems

Subsequence

Subsequence

Subsequence theorems

Cauchy Sequence

Monotonic Sequence

Series

Series

Series Converge

Series Theorems

Function Limits

$\lim_{x \to p} f(x) = q \ (\epsilon - \delta \text{ ball})$

Limit Properties

1. unique
2. $\lim_{x \to p} f(n+m) = \lim_{x \to p} f(n) + \lim_{x \to p} f(m)$
3. algebraic limit theorem:

Continuous Function

f continous at $p \Leftrightarrow \lim_{x \to p} f(x) = f(q)$

Continuous Function on Compact Set I

$f : X \to Y$ is continuous

$\Leftrightarrow \forall$ open set $U \in Y$, $f^{-1}(U)$ is open in X,

$\Leftrightarrow \forall$ closed set $K \in Y$, $f^{-1}(K)$ is closed in X,

Continuous Function on Compact Set II

$f : X \to Y$ is continuous, X compact $\Rightarrow f(x)$ is compact.

Extrem Value Theorem

Uniform Continuity

Intermediate Value Theorm

If $f : [a, b] \to \mathbb{R}$ continuous, and $f(a) < c < f(b)$, then $\exists x \in (a, b)$ such that $f(x) = c$.

Discontinuous

Dirichlet funtion

Simple Discontinuity

Second Discontinuity

Monotonic

Differentiation